Modal Testing and Modal Analysis
**What is Frequency Response Function - FRF**

**Frequency response function** $H(f)$ in the frequency domain and impulse response function $h(t)$ in the time domain are used to describe input-output (force-response) relationships of any system, where signal $a(t)$ and $b(t)$ represent input and output of the physical system. The system is assumed to be linear and time-invariant. Frequency response function and impulse response function are so-called system descriptors. They are independent of the signals involved.

**Convolution:**

$$b(t) = \int h(\tau) \cdot a(t-\tau) \, d\tau = h(t) \cdot a(t)$$

**Multiplication:**

$$B(f) = H(f) \cdot A(f)$$

*Image 1: Explanation of frequency response function*
In the table below you can see typical frequency response function formulations:

<table>
<thead>
<tr>
<th>Dynamic stiffness</th>
<th>Force / Displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receptance</td>
<td>Displacement / Force</td>
</tr>
<tr>
<td>Impedance</td>
<td>Force / Velocity</td>
</tr>
<tr>
<td>Mobility</td>
<td>Velocity / Force</td>
</tr>
<tr>
<td>Dynamic inertia</td>
<td>Force / Acceleration</td>
</tr>
<tr>
<td>Accelerance</td>
<td>Acceleration / Force</td>
</tr>
</tbody>
</table>

The estimation of the frequency response function depends upon the transformation of data from time to the frequency domain. For this computation, we use the Fast Fourier transform (FFT) algorithm which is based on a limited time history. The frequency response functions satisfy the following single and multiple input relationships:

### Single Input Relationship

\[ X_p = H_{pq} F_q \]

Xp is a spectrum of the output, Fp is a spectrum of the input, and Hpq is frequency response function.

### Multiple Input Relationship

\[
\begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_p
\end{bmatrix}_{N_x \times 1} = 
\begin{bmatrix}
H_{11} & \cdots & \cdots & H_{1q} \\
H_{21} & \cdots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
H_{p1} & \cdots & \cdots & H_{pq}
\end{bmatrix}_{N_x \times N_y} 
\begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_q
\end{bmatrix}_{N_y \times 1}
\]

In the image below we can see an example of two inputs - two outputs case.
\[
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} =
\begin{bmatrix}
H_{11} & H_{12} \\
H_{21} & H_{22}
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix}
\]

Image 2: An example of two inputs - two outputs
Modal Test and Modal Analysis in Dewesoftx Software

**Modal test** and analysis are used to determine the engineering structures modal parameters, such as modal frequencies, damping ratios, and mode shapes. The measured excitation and response (or only response) data are utilized in modal analysis, and then dynamic signal analysis and modal parameter identification are processed. The modal test and analysis have been developed for more than three decades, and a lot of progress has been made. It has been widely applied to the engineering field, such as the dynamic design, manufacture and maintenance, vibration and noise reduction, vibration control, condition monitoring, fault detection, model updating, and model validation.

**Modal analysis** is needed in every modern construction. The measurement of system parameters, called modal parameters are essential to predict the behavior of a structure.

These modal parameters are needed also for mathematical models. Parameters like resonant frequency(s), structural damping, and mode shapes are experimentally measured and calculated.

The Dewesoft Modal test module is used for the analysis of e.g. mechanical structures or electrical systems to determine the transfer characteristic (amplitude and phase) over a certain frequency range.

With the small, handy form factor of the Dewesoft data acquisition instruments (DEWE-43, SIRIUS), it is also a smart portable solution for technical consultants coping with failure detection.

The **Modal test module** is included in the Dewesoft X DSA package (along with other modules e.g. Order tracking, Torsional vibration, etc.). Let’s assume there is a mechanical structure to be analyzed. Where are the resonances? Which frequencies can be problematic and should be avoided? How to measure that and what about the quality of the measurement? Probably the easiest way is exciting the structure using a modal hammer (force input) and acceleration sensors for the measurement of the response (acceleration output). At first, the structure is graphically defined in the geometry editor.

Then the points for excitation and response are selected. The test person knocks on the test points while the software collects the data. Next to extracting phase and amplitude, in Analyse mode it is possible to animate the structure for the frequencies of interest. The coherence acts as a measure for the quality. The modal circle provides higher frequency precision and the damping factor.

For more advanced analysis, the data can be exported to several file formats, important is the widely used UNV to read data in e.g. MEScope, ...
Image 3: An example of a modal test on a snowboard
LTI - Linear, Time-invariant Systems

At first, we have to assume that the methods described here apply to LTI (linear, time-invariant) systems or systems that come close to that. LTI systems, from applied mathematics, which appear in a lot of technical areas, have the following characteristics:

- **Linearity**: the relationship between input and output is a linear map (scaled and summed functions at the input will also exist at the output but with different scaling factors)
- **Time-invariant**: whether an input is applied to the system now or any time later, it will be identical

Furthermore, the fundamental result in the LTI system theory is that any LTI system can be characterized entirely by a single function called the system’s impulse response. The output of the system is a convolution of the input to the system with the system’s impulse response.

![Image 4: Linear and time-invariant system](image-url)
**What is Transfer Function**

Transfer functions are widely used in the analysis of systems and the main types are:

- **mechanical** - excite the structure with a modal hammer or shaker (measure force), measure the response with accelerometers (acceleration)
- **electrical** - apply a voltage to the circuit on the input, measure the voltage on the output

For example, in mechanical structures, the transfer characteristics will show dangerous resonances. The frequency range, where the stress of the material is too high, has to be avoided, e.g. by specifying a limited operating range. The simplified process works like that: an input signal is applied to the system and the output signal is measured. The division of response to excitation basically gives the transfer function

\[ H(f) = \frac{Y(f)}{X(f)} \]

In time-domain, this is described in the following way:

\[ x(t) \quad \rightarrow \quad h(t) \quad \rightarrow \quad y(t) = h(t) \cdot x(t) \]

Image 5: Transfer function in the time domain

Laplace transformation leads to the result in the frequency domain:
On the picture below we can see a diagram of Laplace transform, which is often interpreted as a transformation from the time-domain (inputs and outputs are functions of time) to the frequency-domain (inputs and outputs are functions of complex angular frequency), in radians per unit time. Given a simple mathematical or functional description of an input or output to a system, the Laplace transform provides an alternative functional description that often simplifies the process of analyzing the behavior of the system, or in synthesizing a new system based on a set of specifications.
How to obtain the transfer function

1. **Mechanical structure**
   
   - Excite the structure with modal hammer or shaker (measure force)
   - Measure the response with accelerometers (acceleration)

2. **Electrical circuit**
   
   - Apply a voltage to the circuit on the input
   - Measure the voltage on the output of the circuit

Calculate the transfer function between the measured input and output of the system.

Calculate the coherence function. If the coherence is 1 measured response is the power caused totally by the measured input power. If the coherence is less than one at any frequency it indicates that the measured response is greater than due to measured input (additional noise).
Enabling and Adding Modal Test in Dewesoft X

FRF module needs to be enabled in Add module. Click on the Modal Test.

Modal test setup screen appears:

Image 8: Adding new Modal test module

Image 9: The user interface of a Modal test module
Test methods

Depending on the application, Dewesoft offers three different types of setup:

- **Impact hammer** - for excitation an impulse is used (=wide frequency spectrum) - modal hammer
- **Shaker** - the structure is excited by a shaker (or the engine rpm is varied), which sweeps through the frequencies (e.g. 10...1000 Hz)
- **ODS** - operational deflection shapes
Triggered FRF

The easiest test consists of the modal hammer, which is used for exciting the structure with a short impulse (= wide frequency spectrum) and an acceleration sensor measuring the response. The hammer has a force sensor integrated with the tip, the tip ends are interchangeable. For bigger structures, there are big hammers available with more mass to generate a distinct amplitude.

[Video available in the online version]
Trigger parameters

Let's do a short measurement to explain all the parameters. The structure is hit once and the signals are measured.

The hammer signal (upper, blue line) shows a clean shock impact with about 2500 N peak and high damping while the response (lower, red line) starts ringing and smoothly fades out.

**Trigger level**

The Modal test module needs a start criteria in triggered mode, therefore we specify a trigger level of e.g. 2000 N. Each time the input signal overshoots the trigger level, the FRF calculation (FFT window) will start.

**Double hit detection**

However, when the input signal shows multiple impulses after one hit (so-called double hits), Dewesoft X can identify this if you specify a double hit level. When the signal crosses the double-hit-level shortly after the trigger event, you will get a warning.
message and can repeat this point.

Overload level

You can also enable a warning which will be displayed when the hammer impact has exceeded a certain overload level - when the hit was too strong.

The following picture summarizes the different trigger level options.
Now that we have defined the trigger condition, we should ensure that the FRF calculation covers our whole signal to get a good result.

Window length

Let's assume the sample rate of our example is 10 000 Hz and we have set 8192 lines in the FRF setup.

According to Nyquist, we can only measure up to half of the sample rate (5000 Hz) or the other way round, we need at least 2 samples per frequency line. So, our frequency resolution is:

\[
D_f = \frac{10000 \text{ Hz}}{8192 \text{ lines} \cdot 2} = 0.61 \text{ Hz}
\]

The whole FFT window calculation time (window length) is:

\[
t = \frac{1}{D_f} = \frac{1}{0.61} = 1.638 \text{s}
\]
Below you see the cutout data section of the excitation and response signal, which covers pretty much the whole signal.

Note, that the x-axis is scaled in samples (from -819 to 15565, which gives a total of 16384 samples).

\[ 16384 \text{samples} \cdot \left( \frac{1}{10000 \text{Hz}} \right) = 1.63 \text{Hz} \]
Pretrigger

The pretrigger time is set to default by 5%. From the screenshot above you can see that 5% of 16,384 samples is 819 samples, which equals $t_{pre} = 819 \times \left(\frac{1}{10,000 \text{ Hz}}\right) = 81.9$ ms. At sample 0 the trigger occurs.

Image 23: Pretrigger settings
Modal Analysis Data Acquisition System Overview

In most of the cases acceleration sensors, microphones, modal hammers or other force transducers are used for analog input. If they are e.g. voltage or ICP/IEPE type, they are connected directly to the ACC amplifier of the SIRIUS data acquisition system, or DEWE-43/MINITAUR DAQ systems with Dewesoft smart sensor interface DSI adapter (DSI-ACC).

When analog output is needed (for shaker control), the SIRIUS analog out option (8 channels with BNC connector on the rear side of the SIRIUS DAQ system) provides a full-grown arbitrary function generator.
Auto-generated Visual Displays

For an easier start, Dewesoft offers auto-generated displays, which already come with the most often used instruments and an arrangement that makes sense for them according to the type of application.

Dewesoft automatically makes 2 auto-generated displays, one for measurement and one for analysis.

Depending on the measurement type, the display is adjusted: Impact hammer or shaker.

![Image 25: Automatically generated displays]
The excitation and response sections each consist of two 2D graph instruments (scope and FFT) showing array data of hammer and accelerometer signal.

The Info channel will show the current point or events such as double hit.

The Control buttons are used for going from one point to the next or canceling and repeating a point if the result was not satisfying. The OVL display shows if the impact or response signals are too high, exceeding the physical input range of the amplifier.

The FRF Geometry is already animated in the current point during measurement.

Two further 2D graphs on the right side show transfer function and coherence.

**Modal test info channels**

There are additional channels provided by the FRF module, which give status information during the measurement. To display them, please add a Discrete display in Design mode:
The channels Info and OVLChannel can be assigned to it. OVLChannel will only be displayed if the according option has been enabled first.
FRF control channels

- During triggered measurement, after one point is finished, you can continue by pressing the Next point button.
- If you are unsatisfied with the last hit, you can cancel it by using Reject last.
- If all hits for the whole point are incorrect, e.g. if you hit on a point with a wrong number, with Reset point you can delete all the hits done for the current point at once.

All the actions are done using control channels in Dewesoft. These can be modified during measurement. To change it manually, you need to pick the input control display from the instrument toolbar. Set it to Control Channel and Push-button. Channels Reject last, Next point and Reset point can now be assigned from the channel list on the right.

Image 29: FRF control channels
Trigger Parameters

Excitation window length

You can separately adjust the window length of excitation and response in order to reduce the influence of noise appearing after the event of interest. The excitation window length setting is valid for the excitation signal (modal hammer hit). Per default 100% is selected; all of the acquired data will be taken for calculation (all 16,384 samples in our example, the whole shown range).

The excitation FFT is of rectangular window type. In our case the damping is very high (signal fades out quickly), therefore we can select a smaller portion of the signal, e.g. 10% (usually you would define a noise level first to determine it).
The rest of the signal will be cut out completely.

Response window decay

The response FFT is an exponential window type. When the response signal is fading out slowly (low damping), the user can specify a certain time after which the signal is faded to zero (exponential decay function). This helps to reduce noise at low amplitudes and shortens the measurement time. The picture below shows how the response window is decayed when different percentages are selected in Dewesoft.
Averaging of hits

The result can be improved by averaging the excitation and response spectrum over a number of impacts. Therefore, the first e.g. 5 hits will be recognized and taken into calculation, then you move on to the next point.

Image 33: Averaging of hits

Image 34: Settings for averaging hits
Impact hammer, 1 point excitation

When all acceleration sensors are mounted, the structure is excited at one point by the modal hammer (average over a number of hits can also be done of course).

Image 35: Excitation in one point with an impact hammer

Image 36: Excitation in one point with an impact hammer setup
Impact hammer, roving excitation sensor

1 roving excitation, 1 response

In this operation mode, there is one acceleration sensor mounted in a fixed position on the structure. The modal hammer is moving through the points (e.g. doing 5 hits in each point, which are averaged). This is the easiest test and requires only one hammer and one sensor.
Image 38: Roving excitation with an impact hammer setup
Impact hammer, roving acceleration sensor

The hammer is always exciting the structure in the same position. Now the acceleration sensor is moved to different positions. The disadvantage of this setup is, that the mass of the acceleration sensor changes the structure differently in every point, therefore, influences the measurement (this effect is called mass loading). Also between each measurement, the sensor has to be mounted again, which results in a lot of work.
Free run (sweep)

When doing a frequency sweep and measuring the responses, you have the advantage that the coherence will be much better over the whole frequency range compared with a triggered setup. Of course, you are facing a more extensive setup in terms of hardware, you’ll probably need a shaker (and a shaker controller, which keeps the amplitude constant over the frequency range). The channel setup of a typical shaker modal test is shown below.

The FFT windowing section is similar to the impact hammer FRF. You should ensure that the sweep is slow enough because the FFT needs some time for calculation (number of lines, resolution). Again, on the left, we have the excitation and on the right side response channels. If you enable the Use function generator checkbox, the FGEN settings Waveform, Start frequency and Stop frequency and the AO channel column in the excitation section will also be visible. These settings are the same as in the Analog out section (function generator).
Furthermore, you can adjust here the sweep time and amplitude/phase settings, if you enter the Setup of the according to the channel (AO 1 in our example). On the right side, you can tick the checkbox Show info channels, e.g. seeing the current frequency during sweep is very helpful.

When you switch to Measure mode or press the Store button, the sweep will start. In comparison with the triggered measurement, our excitation(s) and response(s) will in most of the cases consist now of sine waves, with distinct amplitude and phase shift.
When using a sine sweep, as the sweep moves through the frequencies, the bode plots will be updated. Putting the AO/Freq channel on a separate display is a good way to show the current frequency.
The picture above shows two 2D graph instruments with transfer functions 2-1 and 3-1 (amplitude on top and the phase below) during a sweep. The left side is already calculated while the right side is ongoing.
The usual application for the free-run option is on a shaker. If the shaker is externally controlled, we can measure back the excitation signal (with a force sensor) and use it as a reference. Of course, it would also be possible to use an engine instead of the shaker and analyze the transfer functions during runup or coast down.
If we tick Use function generator, the FRF module accesses the FGEN section (requires Analog output option on the Dewesoft instrument (AO)). It generates now e.g. a sine sweep from 10 to 1000 Hz. The shaker controller guarantees a defined amplitude over all frequencies. With the force sensor, we measure back the excitation force. Please consider that Dewesoft will not do the shaker control (control loop for amplitude), because of speed limitations. Practically a shaker control device (shaker control box in the above picture) will be used in between.
ODS - Operational deflection shapes

[Video available in the online version]
In the experimental modal analysis (EMA), the structures are excited by artificial forces and both the inputs (excitation) and outputs (response) are measured to get the frequency response functions (FRF) or impulse response functions (IRF) by digital signal processing. Modal parameters can be identified from FRF or IRF by identification algorithms in the frequency domain or the time domain. EMA tests are usually carried out in the lab, with the advantage of high signal to noise ratio (SNR) and easy to change test status.

EMA identification methods can be classified into a time domain (TD) methods and frequency domain (FD) methods according to different identification domain. Also, they can be classified according to a different number of input and output:

- SISO (single input single output),
- SIMO (single input multiple outputs),
- MIMO (multiple inputs multiple outputs).

The FRF is generally utilized for the EMA in the frequency domain, which is estimated from the excitation and response signals. Then the modal parameters are identified by constructing the parametric or nonparametric models of the FRF and curve fitting them. The IRF is generally utilized for the EMA in the time domain. It can be obtained from the inverse FFT of FRF.

Time domain methods are suitable for the global analysis in a broad frequency band, which have good numeric stability. However, there are some limitations too:

- very difficult to confirm the order of math model,
- always time-consuming,
- many calculation modes got with the structural modes and difficult to delete them,
- many settings needed, complicated-to-use,
- not being able to take into account the influence of out-band modes.

On the opposite side, frequency domain methods are always reliable, rapid, easy-to-use, with the capacity to consider the out-band modes and analysis uneven spaced FRFs, so they are applied widely.
OMA - Operational Modal Analysis

Operational modal analysis is used for large civil engineering structures, operating machinery or other structures, making use of their output response only. These structures are always loaded by natural loads that cannot easily be controlled and measured, for instance, waves load (offshore structures), the wind loads (buildings) or traffic loads (bridges).

Compared with EMA, OMA has its outstanding advantages. In OMA, the structure studied, is excited by natural loads instead of some expensive excitation equipments as used in EMA. In fact, it is very difficult to excite large structures by artificial means. So OMA is more economic and fast, and endowed by nature with characteristics of multiple-input/multiple-output (MIMO). It could be used to distinguish closely coupled modes. Moreover, all the measured responses come under operational state of structures, and their real dynamic characteristics in operation could be revealed, so OMA is very suitable for health monitoring and damage detection of large-scale structures.
MIMO - multiple shaker excitation

Multiple-Input Multiple-Output (MIMO) measurement techniques are a well-proven and well-established method for collecting FRF data sets. MIMO methods offer some distinct advantages for the measurement and extraction of basic modal parameters especially while testing larger structures.

The main advantage of MIMO is that the input-force energy is distributed over more locations on the structure. This provides a more uniform vibration response over the structure, especially in cases of large and complex structures and structures with heavy damping. In order to get sufficient vibration energy into these types of structures, there is a tendency to overdrive the excitation DOF when only a single shaker is used. This can result in non-linear behavior and deteriorates the estimation of the FRFs. Excitation in more locations often also provides a better representation of the excitation forces that the structure experiences during real-life operation.

The response transducers must be roved around unless there are sufficient transducers available to cover all the response DOFs. With this type of testing, uncorrelated random (continuous, burst, or periodic random) excitation signals are used. Burst random and periodic random signals have the ability to provide leakage-free estimates of the FRFs, i.e., without resolution-bias errors, which is an advantage compared to continuous random signals.

A demo measurement was done on a plane model. The structure was excited by two shakers with uncorrelated excitation signals. The responses were measured with 3-axial accelerometers and the excitation signals were measured with force transducers. The voltage signal for the shakers was supplied from the Dewesoft Function generator (AO module).

On the left side of the Modal test UI, you select Shaker as a test method, resolution of the measurement, and additional output channels.

On the right side of the UI, define the excitation source and excitation channels. We will use Dewesoft AO to drive the shakers and we select Burst random noise as the type of excitation.
We will measure the excitation with 2 force transducers in Z+ direction, at points 1 and 2.

We measured the responses with 4 tri-axial accelerometers in X, Y, and Z direction (points 1, 2, 4, 5) and with 2 uni-axial accelerometers in Z direction (points 3 and 6). Setup was done accordingly, as shown in the image below.
On time-domain recorder (top right corner) you can see time domain channels from excitation and responses.

On the left side, the results are displayed on 2D graphs. You can see transfer functions, coherences, and MIF (mode indicator functions). The geometry is animated from the selected frequency.

Image 55: MIMO results
Geometry editor

In Dewesoft X, you can quickly draw simple structures, as well as import more complex ones. Cartesian and cylindrical coordinate systems are supported, which is great for drawing circular objects.

The index numbers defined in the channel setup before are used as Point numbers in the geometry for animation.

[Video available in the online version]
Importing a CAD file into Dewesoft X's geometry editor

It has always been a big question when it comes to importing a more complex geometry into the Dewesoft X's FRF geometry editor for modal analysis.

It seems not many CAD software support the export to UNV; actually, it's a process of finite element meshing, calculating the polygons.

Here you can find a freeware-converter GMSH from e.g. STL-to-UNV (http://geuz.org/gmsh/).

You can download the program by clicking on a link: http://geuz.org/gmsh/bin/Windows/gmsh-2.8.5-Windows.zip

After downloading it, open the program. Then you can open any STL file.

The geometry will be seen in GMSH program.
The click on Save as and choose *.unv file format and rename the file to .unv.

After that, the importing of geometry in Dewesoft X in the same as it is described on the previous page.

All the nodes and triangles are defined in the .unv file format.
Geometry is now ready to be used for modal animation.
FRF - frequency response function

For the following explanation of parameters, a triggered FRF was done on a snowboard structure. All 39 excitation points were sequentially hit by the modal hammer and related to 1 accelerometer placed in the center.

Only 1 hammer and 1 sensor were used!

From the channel list on the right side, we see that each point (#1, #2, #3, #4) is related to the reference point (#1). For each excitation point, a transfer function was calculated, e.g. TF_1Z+/3Z+.

A transfer function consists of amplitude and phase part or real and imaginary parts. The 2D graph is the instrument to use, there you can select what you want to display by using the properties from the left side.
To make a bode plot, use two 2D graphs below each other. The above one shows the amplitude (y-axis type: LOG), the lower one the phase (y-axis type: LIN).

When the amplitude of the transfer function shows a local maximum, and the phase is turning at this point, it usually indicates a resonance. But to avoid an erroneous statement, other parameters have to be checked as well!
Coherence

Coherence is used to check the correlation between the output spectrum and the input spectrum. So you can estimate the power transfer between input and output of a linear system. It shows how well the input and output are related to each other.

Autospectrum

Autospectrum is a function commonly explored both in signal and system analysis. It is computed from the instantaneous (Fourier) spectrum as:

![Image 74: Autospectrum](image)

There is a new, fundamental function - cross-spectrum - in the dual-channel processing. It is computed from the instantaneous spectra of both channels. All other functions are computed during post-processing from the cross-spectrum and the two auto spectrums - all functions are the functions of frequency.

Cross spectrum

Based on complex instantaneous spectrum $A(f)$ and $B(f)$, the cross-spectrum $S_{AB}$ (from $A$ to $B$) is defined as:
The amplitude of the cross-spectrum $S_{AB}$ is the product of amplitudes, its phase is the difference between both phases (from A to B). Cross spectrum $S_{BA}$ (from B to A) has the same amplitude, but opposite phase. The phase of the cross-spectrum is the phase of the system as well.

Both auto spectra and cross-spectrum can be defined either as two-sided (notation $S_{AA}$, $S_{BB}$, $S_{AB}$, $S_{BA}$) or as one-sided (notation $G_{AA}$, $G_{BB}$, $G_{AB}$, $G_{BA}$). One-sided spectrum is obtained from the two-sided one as:

\[
A(f) = |A(f)| \cdot e^{i\Phi_A(f)} \\
B(f) = |B(f)| \cdot e^{i\Phi_B(f)} \\
S_{AB}(f) = E\left[|A(f)| \cdot |B(f)| \cdot e^{i[\Phi_B(f) - \Phi_A(f)]}\right]
\]
The cross-spectrum itself has little importance, but it is used to compute other functions. Its amplitude $|G_{AB}|$ indicates the extent to which the two signals correlate as the function of frequency and phase angle $\angle G_{AB}$ indicates the phase shift between the two signals as the function of frequency. The advantage of the cross-spectrum is that influence of noise can be reduced by averaging. That is because the phase angle of the noise spectrum takes random values so that the sum of those several random spectra tends to zero. It can be seen that the measured auto spectrum is a sum of the true auto spectrum and auto spectrum of noise, whilst the measured cross-spectrum is equal to the true cross-spectrum.
Coherence

Coherence function $\gamma^2(f)$ indicates the degree of a linear relationship between two signals as a function of frequency. It is defined by two auto spectra (GAA, GBB) and a cross-spectrum (GAB) as:

$$\gamma^2(f) = \frac{|G_{AB}(f)|^2}{G_{AA}(f) \cdot G_{BB}(f)}$$

At each frequency, coherence can be taken as a correlation coefficient (squared) which expresses the degree of the linear relationship between two variables, where the magnitudes of auto spectra correspond to variances of those two variables and the magnitude of cross-spectrum corresponds to covariance.
The coherence value varies from zero to one. Zero means no relationship between the input A and output B, whilst one means a perfectly linear relationship.

\[ 0 \leq \gamma^2(\xi) \leq 1 \]

There are four possible relationships between input A and output B:

<table>
<thead>
<tr>
<th>Perfectly linear relationship</th>
<th>A sufficiently linear relationship with a slight scatters caused by noise</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Image 78: Linear" /></td>
<td><img src="image" alt="Image 79: Sufficiently linear" /></td>
</tr>
<tr>
<td><img src="image" alt="Image 80: Non-linear" /></td>
<td><img src="image" alt="Image 81: No relationship" /></td>
</tr>
</tbody>
</table>

Low values indicate a weak relation (when the excitation spectrum has gaps at certain frequencies), values close to 1 show a representative measurement.

That means when the transfer function shows a peak, but the coherence is low (red circles in the picture below), it must not necessarily be a real resonance. Maybe the measurement has to be repeated (with different hammer tips), or you can additionally look for the MIF parameter.
Coherence is a Vector channel and therefore displayed with a 2D graph instrument.

The coherence is calculated separately for each point (e.g. Coherence_3Z/1Z, Coherence_4Z/1Z).

In the case of no averaging, coherence is always equal to 1. In the case of averaging and samples, GAB influenced by noise, deviations in the phase angles cause that the resulting magnitude |GAB| is lower than it would be without the presence of noise (see the picture below). The presence of non-linearity has a similar influence.

Image 82: Coherence displayed on a 2D graph in Dewesoft

Image 83: Averaging with and without noise
Measurement screen - video

[Video available in the online version]
Mode indicator function (MIF)

If all parts of a structure are moving sinusoidally with the same frequency (fixed phase relations), this motion is called normal mode. This happens at resonance or natural frequencies. Depending on the structure, material, and bounding conditions there exist a number of mode shapes (e.g. twisting, bending, half-period, full-period movement...).

These are usually found out by finite element simulation software, or by experimental measurement and analysis.

When the amplitude of the transfer function shows a local maximum, and the phase is turning at this point, it usually indicates a resonance. To be sure, also the Coherence should be checked as described before. And last, you can look for the MIF (=Mode Indicator Function).

A MIF close to 1 indicates a mode shape.

The spikes shown in the picture below are very likely resonance frequencies. Just click on them and check the movement in the geometry instrument.

MIF is a Vector channel, and therefore also displayed with a 2D graph instrument.

The MIF is calculated from all transfer functions (all points), therefore, is only one channel.
Structure movement animation

The structure movement animation is done by putting sine functions with the amplitudes and phases from the measurement into the geometry model points. The animation is done in one direction (in our example Z+). You can animate the structure for a single frequency, which can be chosen in the 2D graph when setting the Cursor type to Channel cursor, as shown below. All FRF instruments will follow the channel cursor.

[Video available in the online version]
Modal circle

Finally, when you are certain that the point you are looking at is a resonance, you might want to get its exact frequency and damping factor. As the FFT can never be that precise (high line resolution needs long calculation time, which is not given when there is a hammer impact), there are some mathematical methods to interpolate.

[Video available in the online version]
Export of complex data

After the measurement is done the data can be exported to a lot of different file formats, e.g. UNV/UFF, Diadem, Matlab, Excel, Text... The transfer function can be separately exported by Real, Imag, Ampl or Phase part, whatever you prefer.

[Video available in the online version]

![Image 94: Export complex channels]

In MS Excel, for example, the transfer function data will appear on a sheet called Single value. For each transfer function, Real/Imag/Ampl/Phase is exported.

If you prefer it differently, data rows and columns can simply be exchanged in MS Excel by copying and using the Transpose function from the submenu when pasting.
Image 95: Export of complex data to MS Excel
Export in UNV / UFF format

The Universal File Format (also known as UFF or UNV format) is very common in modal analysis. Depending on the header, it can contain either transfer functions, coherence, geometry, ... or various other data.

The following example shows how to export data recorded by Dewesoft into Vibrant Technologies ME Scope analysis software and how to display it there.

First, choose the UNV export from the export section and the option Export complex channels if you want to export phase, real and imaginary part. Then select all your transfer functions (you can use the Filter and type TF for simplification). When clicking on the Export button you will create a UNV datafile.

1. In FRF, the geometry editor saves the structure also in UNV format. This creates the UNV geometry file.

2.
3. Start a software that can import UNV files (like ME Scope, N-Modal, ...) and click File → Import Data block. Select the UNV datafile.

4. The transfer functions are already recognized.

5. The transfer functions are already recognized.
8. Then click File Import Structure and select the UNV geometry file.

9. Now both data and geometry are successfully imported. Let's try to animate it, select Draw Animate Shapes.
13. Image 101: Animate the structure

14.

15. A pop up appears, and we select to match the structure and transfer data. Equations are created.

16. Finally, you can select a peak on a transfer function and enjoy the animation.

17.
Image 104: Animation of a structure at a selected peak
Example - Impact hammer measurement

As the triggered measurement might be difficult to understand, this section shows how to use the mentioned controls and tools step-by-step.

Let’s say we want to analyze this metal sheet structure. At first, we define the direction of analysis (orientation up/down, Z-axis), then we put it on a soft rubber foam that it can vibrate freely. Of course, hanging it with rubber bands from the roof would be better but would also take more time to wait for each point until the ringing fades out.

Then mark equidistant points, in our case from #1 to #24. The higher the number of points, the more detailed the animation will be. It is also helpful to write numbers next to the points. They should be consistent with the structure, channel setup, and FRF geometry in software.

[Video available in the online version]
Example - excitation of a structure with a shaker

This is a practical example showing shaker mode FRF. The Analog out of the SIRIUS instrument (Function Generator) is connected to an audio amplifier which drives a loudspeaker. On the membrane, a metal structure (metal beam) is mounted on a force transducer (excitation) and two acceleration sensors (responses).

1. In the Analog section, we define our force sensor and the two accelerometers. They are all of IEPE type. As we want to analyze our structure up to 1000 Hz, we select a sampling rate of e.g. 5000 Hz.

2.

3. Next, we add a Modal test module (Shaker measurement type) and choose the Use function generator. A window size of 1024 lines results in a nice resolution of 2.44 Hz. We select a sine sweep from 1 to 1000 Hz. The index numbers 1, 2, and
3 are entered according to the structure, the direction is Z+ for all.

1. Also, check the Function generator section. Start and stop frequency are already copied from the Modal test module. We adjust the sweep time (120 seconds) and amplitude (1 V) for now. Startup time and the fall time is 0.1 s by default, which prohibits sudden crackles, that could result in wide-spectrum noise at the beginning and end of the measurement.

Now we are ready for drawing the structure. Go to measure mode, the screen Modal test should be autogenerated. Click on the FRF geometry instrument and select Editor from the left side. Then add 3 points with the + button, example coordinates as shown below. Then save the structure by clicking on File Save UNV...
Now we are ready for measurement. When you click the store button, the Function generator will start, the AO will sweep from 1 to 1000 Hz. The transfer functions will smoothen from left to right side, here you see a snapshot currently at 357 Hz.

Image 119: Defining the geometry for demo measurement
Finally, we can look at the result. The coherence of both channels related to the excitation looks very nice. The green line (MIF) indicates mode shapes, click on the peaks and the structure will be animated.

Image 120: Measurement screen during a shaker demo measurement
Image 121: Reviewing of measurement results in Analyse mode